2020−2021 AUT Admission Test
Mathematics (Sample)

< **Multiple choice Questions** > There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet. For each correct answer, you will get the points indicated next to each question number. No penalty point is applied to an incorrect answer.

(Sample)

1. [4 points] How many 3 digit natural numbers are such that each digit is either 4 ,5, 6 or 7 and all three digits are distinct? (For example, 456 is one of such numbers.)

① 20 ② 21 ③ 22 ④ 23 ⑤ 24

< **Short answer questions** > Mark your answer on the OMR answer sheet as example. There is only one correct answer per each question. Mark your answer choice on the OMR answer sheet. ex) If the answer is 538, mark ⑤, ③, ⑧ as below. If the answer is 17, mark ⓪, ①, ⑦.

100	**10**	**1**
⓪	⓪	⓪
①	①	①
②	②	②
③	●	③
④	④	④
●	⑤	⑤
⑥	⑥	⑥
⑦	⑦	⑦
⑧	⑧	●
⑨	⑨	⑨

(Sample)

15. [10 points] Consider a non-decreasing sequence of positive integers 1,3,3,3,5,5,5,5,5,7,7,7,7,7... in which the integer (2n+1) appears (2n+1) times. Calculate the remainder when the 2019th term is divided by 10,

< Multiple choice Questions >

1. [4 Points] Let $3^x = 2020$. Simplify

$$|x-6|+|x-8|.$$

① $2x-14$ ② $14-2x$ ③ 2

④ -2 ⑤ $2-x$

2. [4 Points] Evaluate $\log_2 9 \times \log_3 4 \times \sqrt[3]{27}$.

① 4 ② 6 ③ 8 ④ 10 ⑤ 12

3. [4 Points] Let a, b be the two solutions of the quadratic equation $3x^2+4x-3=0$. Find the value of $a \times b$.

① -3 ② -1 ③ 1 ④ 3 ⑤ 4

4. [6 Points] Find the sum of all integers satisfying

$$x^2-3x \le 4.$$

① 6 ② 7 ③ 8 ④ 9 ⑤ 10

5. [6 Points] Find the value of

$$\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\frac{1}{4\cdot 6}\cdots+\frac{1}{9\cdot 11}$$

① $\frac{18}{55}$ ② $\frac{36}{55}$ ③ $\frac{72}{55}$ ④ $\frac{93}{55}$ ⑤ $\frac{10}{11}$

6. [6 Points] Find the value of

$$\sin(\text{arctg } 2-\text{arctg}\frac{1}{2}).$$

① $\frac{3}{4}$ ② $\frac{2}{5}$ ③ $\frac{3}{5}$ ④ $\frac{4}{5}$ ⑤ $\frac{5}{6}$

7. [6 Points] If the tangent line to the curve $y=x^3-2x^2+2$ at the point $(1,1)$ passes through $(0,a)$, find the value of a.

① 1 ② $\frac{3}{2}$ ③ 2 ④ $\frac{5}{2}$ ⑤ 3

8. [6 Points] If $\sin\theta\cos\theta=-\frac{4}{9}$, find the value of $\frac{\sin^2\theta}{(1+\tan\theta)^2}$.

① $\frac{5}{9}$ ② $\frac{7}{9}$ ③ $\frac{11}{9}$ ④ $\frac{16}{9}$ ⑤ $\frac{25}{9}$

9. [6 Points] If $a+b=3$, $ab=1$ and $a>b$, calculate the value of a^2-b^2.

① $\sqrt{5}$ ② 2 ③ $4\sqrt{3}$ ④ $3\sqrt{5}$ ⑤ 9

10. [6 Points] A function on the real numbers given by

$$f(x)=\begin{cases} e^{ax}, & x<0 \\ -bx+c, & x\ge 0 \end{cases}$$

is differentiable at $x=0$. Find $a+b+c$.

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

11. [6 Points] Evaluate the following limit

$$\lim_{n\to\infty}\left(\frac{n^3}{1^2+2^2+3^2+\cdots+n^2}\right)$$

① 0 ② 1 ③ 2 ④ 3 ⑤ 4

12. [6 Points] Suppose that a function $f(x)$ satisfies $(x^2+1)f(x)=xf(x-1)+3$ for every real number x. Find the value of $f(1)$.

① 2 ② 3 ③ 4 ④ 5 ⑤ 6

13. [6 Points] Evaluate $\int_1^2 x\sqrt{x^2-1}dx$.

① $\sqrt{2}$ ② $\sqrt{3}$ ③ $2\sqrt{2}$ ④ $2\sqrt{3}$ ⑤ $5\sqrt{2}$

14. [6 Points] Suppose that $\log_{27}\sqrt{a}=\log_3 b^2$ with $a,b>0$. Find the value $\log_b a$.

① 3 ② 9 ③ 12 ④ 27 ⑤ 81

15. [6 Points] Let

$$\sum_{n=1}^{2020}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^n = a+bi.$$

Find the value of $a \times b$.

① $-\frac{\sqrt{3}}{2}$ ② $-\frac{\sqrt{3}}{4}$ ③ $\frac{\sqrt{3}}{4}$ ④ $\frac{\sqrt{3}}{2}$ ⑤ $\frac{3\sqrt{3}}{2}$

16. [6 Points] How many 3 digit natural numbers are such that each digit is either all digits are odd or all digits are even?

① 225 ② 250 ③ 325 ④ 350 ⑤ 500

17. [8 Points] If $f(x) = ax^4 + bx^3 + cx^2 + dx$ satisfies

$$\lim_{x\to\infty}\frac{f(x)+f(-x)}{x^2} = 2,$$

find the value of $\int_{-1}^{1} f(x)dx$.

① $-\frac{1}{2}$ ② 0 ③ $\frac{1}{2}$ ④ $\frac{2}{3}$ ⑤ $\frac{3}{4}$

18. [8 Points] Find the indefinite integral of

$$\frac{1}{x\ (\ln x)(\ln\ln x)}$$

① $\ln\ln x$ ② $\ln\ln\ln x$ ③ $-\frac{1}{\ln\ln x}$

④ $-\frac{1}{\ln\ln\ \ln\ x}$ ⑤ $-\frac{1}{2(\ln\ln x)^2}$

19. [8 Points] Find the slope of the tangent line to the curve $y^3 = \ln(5-x^2) + 2xy - 3$ at $(2,1)$.

① $-\frac{1}{2}$ ② $\frac{1}{2}$ ③ 1 ④ 2 ⑤ $\frac{5}{2}$

20. [8 Points] Let $f(x) = x^4 - x^3 + x^2 - x + 1$ and $g(x)$ be a differentiable function. Let $h(x) = g(f(x))$. If $h'(0) = 5$, find the value of $g'(1)$.

① -5 ② -4 ③ -3 ④ -2 ⑤ -1

21. [8 Points] Find the area of the region bounded by three curves $y = x^2$ and $y = \sqrt{x}$

① $\frac{1}{12}$ ② $\frac{1}{6}$ ③ $\frac{1}{4}$ ④ $\frac{1}{3}$ ⑤ $\frac{1}{2}$

22. [8 Points] Suppose that a continuous function $f(x)$ satisfies $(e^x - 1)f(x) = \sin x + a$ for every real number x. Find the sum $a + f(0)$.

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

23. [8 Points] Evaluate the following limit.

$$\lim_{x\to 0}\frac{\sin x - \tan x}{x^3}$$

① -1 ② $-\frac{1}{2}$ ③ $\frac{1}{2}$ ④ 1 ⑤ 2

24. [8 Points] Let $y = ax + b$ and $y = cx + d$ be two tangent lines to the curve $f(x) = x^2 + 4$ passing through the origin. Find the value of ac.

① -1 ② -25 ③ -16 ④ 16 ⑤ 25

25. [8 Points] Suppose that $a + b + c + d = 1$ and $a, b, c, d \geq 0$. Find the maximum value of $ab + bc + cd + da$.

① $\frac{1}{10}$ ② $\frac{1}{8}$ ③ $\frac{1}{6}$ ④ $\frac{1}{4}$ ⑤ $\frac{1}{2}$

26. [8 Points] Let $f(x)$ be a differentiable function and $g(x) = \frac{f(x)}{e^{2x}+1}$. If $f(0) - f'(0) = 1$, find the value of $g'(0)$.

① $-\frac{1}{2}$ ② $-\frac{1}{4}$ ③ -1 ④ $\frac{1}{4}$ ⑤ $\frac{1}{2}$

27. [8 Points] Suppose that a function $y = f(x)$ is differentiable and satisfies

$$f(x) = 3x^2 + x\int_0^1 f(x)dx.$$

Find the value of $f(1)$.

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

< Short answer questions >

28. [6 Points] Evaluate the following sum.

$$\sum_{n=1}^{30} (-1)^n n^2$$

29. [8 Points] Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ and $A^9 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find the value of a.

30. [8 Points] Find the number of all integers $x > 0$ for which the limit exists.

$$\lim_{n\to\infty} \left(\frac{(\log_5 x)^n}{3^n + 2^n} \right)$$