2021-2022 AUT
Admission Examination

Mathematics

SAMPLE

Test ID Number	
Full Name	
Major	

Multiple choice questions

1. [**2** points] If $5\sin x = 4$ with $\frac{\pi}{2} < x < \pi$, then $\cos x$ is equal to

① $-\frac{3}{5}$ ② $-\frac{3}{4}$ ③ $\frac{1}{3}$ ④ $\frac{3}{4}$ ⑤ $\frac{3}{5}$

2. [**2** points] Find the sum of all **INTEGERS** x satisfying the following inequality:

$$x^2 - 2x \leq 2$$

① -1 ② 0 ③ 1 ④ 2 ⑤ 3

3. [**2** points] Which of the following is the **LARGEST**?

① $\sqrt{3}$ ② $\sqrt[3]{3\sqrt{2}}$ ③ $\sqrt{2\sqrt[3]{3}}$ ④ $\sqrt[3]{5}$ ⑤ $\sqrt[6]{23}$

4. [**3** points] If $\omega^2 - \omega + 1 = 0$, then $\omega^{2021} - 2\omega^{2022} + 3\omega^{2023}$ is equal to

① 1 ② $2\omega - 1$ ③ $2\omega + 1$

④ ω^2 ⑤ $2\omega^2$

5. [**3** points] If α and β are the roots of the equation $(\log_3 x)^2 - \log_3 x^3 = 9$, then $\alpha\beta$ is equal to

① 27 ② 18 ③ 9 ④ 6 ⑤ 2

6. [**3** points] Let $2^x = 2022$. Simplify the following:

$$|x - 9| + 2|x - 10| + |x - 11|$$

① 9 ② $2x - 18$ ③ $2x$

④ $9 - 2x$ ⑤ $18 - 2x$

7. [**3** points] Simplify the following:

$$\sin\left(\frac{\pi}{12}\right) \times \sin\left(\frac{5\pi}{12}\right)$$

① $\frac{\sqrt{3}}{4}$ ② $\frac{1}{4}$ ③ $\frac{\sqrt{2}}{4}$ ④ $-\frac{1}{4}$ ⑤ $-\frac{\sqrt{3}}{4}$

8. [**3** points] If $a-b=4$ and $ab=-2$, then a^3-b^3 is equal to

① 10 ② 20 ③ 30 ④ 40 ⑤ 50

9. [**3** points] Simplify the following:

$$\sin^2\left(\frac{\pi}{20}\right)+\sin^2\left(\frac{\pi}{10}\right)+\sin^2\left(\frac{3\pi}{20}\right)+\sin^2\left(\frac{\pi}{5}\right)+\sin^2\left(\frac{\pi}{4}\right)+\sin^2\left(\frac{3\pi}{10}\right)+\sin^2\left(\frac{7\pi}{20}\right)+\sin^2\left(\frac{2\pi}{5}\right)+\sin^2\left(\frac{9\pi}{20}\right)$$

① 3 ② $\frac{7}{2}$ ③ 4 ④ $\frac{9}{2}$ ⑤ 5

10. [**3** points] Simplify the following:

$$\log_7\left(1-\frac{1}{2}\right)+\log_7\left(1-\frac{1}{3}\right)+\log_7\left(1-\frac{1}{4}\right)+\cdots+\log_7\left(1-\frac{1}{49}\right)$$

① -4 ② -2 ③ 0 ④ 2 ⑤ 4

11. [**3** points] Evaluate the following limit:

$$\lim_{x\to 0}\frac{1-\cos x}{x}$$

① -2 ② $-\frac{1}{2}$ ③ 0 ④ $\frac{1}{2}$ ⑤ 2

12. [**3** points] Find the **MINIMUM** value of $\frac{x^2}{x-3}$ for $x>3$.

① 0 ② 3 ③ 6 ④ 9 ⑤ 12

13. [**3** points] Let $A=\begin{pmatrix}1 & a\\ 0 & 2\end{pmatrix}$. If $A^2=\begin{pmatrix}b & 9\\ c & d\end{pmatrix}$. Then, $a+b+c+d$ is equal to

① 2 ② 4 ③ 6 ④ 8 ⑤ 10

14. [**3** points] A differentiable function $f(x)$ defined on the real line has the following values:

x	-1	0	2
$f(x)$	1	7	3
$f'(x)$	4	1	-2

Find $g'(1)$ for $g(x)=\big(f(2x)\big)^3$.

① -108 ② -36 ③ -12 ④ 12 ⑤ 36

15. [**3** points] Suppose the following holds for some real numbers a and b.

$$\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)^{161} = a + b\,i$$

Then, ab is equal to

① $-\frac{1}{2}$ ② $-\frac{1}{\sqrt{2}}$ ③ $\frac{\sqrt{3}}{4}$ ④ $\frac{1}{2}$ ⑤ $\frac{1}{\sqrt{2}}$

16. [**3** points] If $\mathbb{P}(A|B)=\frac{1}{5}$, $\mathbb{P}(B|A)=\frac{1}{2}$, and $\mathbb{P}(A\cup B)=\frac{1}{4}$, find $\mathbb{P}(A\cap B)$.

① $\frac{1}{6}$ ② $\frac{1}{12}$ ③ $\frac{1}{15}$ ④ $\frac{1}{18}$ ⑤ $\frac{1}{24}$

17. [**3** points] Find the **MINIMUM** value of $x+2y$ where $x^2+y^2=1$.

① -2 ② $-\sqrt{5}$ ③ $-\sqrt{3}$ ④ $-\sqrt{2}$ ⑤ -1

18. [**3** points] Suppose that a differentiable function f satisfies

$$f(x)=\sin x + \int_0^{\pi}\left(f'(t)\right)^2 dt$$

for all x. Then, $f(\pi)$ is equal to

① $\frac{\pi}{3}$ ② $\frac{\pi}{2}$ ③ π ④ 0 ⑤ 1

19. [**3** points] A quadratic function $y=f(x)$ satisfies $f(0)=1$ and

$$\int_{-1}^{2} f(x)dx = \int_{-1}^{0} f(x)dx = \int_{0}^{2} f(x)dx$$

Then, $f(-2)$ is equal to

① -7 ② -6 ③ -5 ④ -4 ⑤ -3

20. [**3** points] Evaluate the following limit:

$$\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{-2n}$$

① e^2 ② e ③ 1 ④ e^{-1} ⑤ e^{-2}

21. [**4** points] Find the area of the triangle ΔABC with sides $\overline{AB} = 7$, $\overline{BC} = 4$, and $\overline{AC} = 5$.

① $2\sqrt{3}$ ② $2\sqrt{6}$ ③ $2\sqrt{7}$ ④ $4\sqrt{6}$ ⑤ $4\sqrt{7}$

22. [**4** points] Find the area of the region that is enclosed by the curves $y = 2x$ and $y = x^2$.

① $\frac{8}{3}$ ② $\frac{4}{3}$ ③ $\frac{7}{6}$ ④ 1 ⑤ $\frac{5}{6}$

23. [**4** points] Consider the function $f(x)$ defined by

$$f(x) = \lim_{n\to\infty} \frac{2^{n-1}\sin^{2n+1}(2x) + \frac{\pi}{6} - x}{2^n \sin^{2n}(2x) + 1}$$

Then, $(f \circ f)(0)$ is equal to

① 0 ② $\frac{\sqrt{3}}{4}$ ③ $\frac{\sqrt{3}}{2}$ ④ $\frac{1}{2\sqrt{2}}$ ⑤ $\frac{1}{4\sqrt{2}}$

24. [**4** points] Find the **LARGEST** real number k such that $f(x) = -x^3 + kx^2 - 3kx + 1$ satisfies

$f(x_1) > f(x_2)$ whenever $x_1 < x_2$

① 3 ② 5 ③ 7 ④ 9 ⑤ 11

25. [**4** points] Suppose the function defined by

$$f(x) = \int_0^x e^{t^2}\, dt$$

satisfies $f(a) = \frac{\pi}{2}$ for some constant a. Then,

$$\int_0^a \sin\big(f(x)\big)\, e^{x^2}\, dx$$

is equal to

① $\frac{1}{\pi}$ ② $\frac{1}{2}$ ③ 1 ④ 2 ⑤ π

Short answer questions

26. [**4** points] Find the sum of all x with $5 \leq x \leq 500$ such that $\log_{10} x$ is an integer.

27. [**4** points] Evaluate the following integral:

$$\int_1^4 x\sqrt{17-x^2}\ dx$$

28. [**5** points] Let α be the sum of **ALL** solutions to the trigonometric equation

$$\cos 2x - \cos x + 1 = 0, \qquad 0 \leq x \leq \pi.$$

Evaluate

$$\frac{72\,\alpha}{\pi}$$

29. [**5** points] Find the real number k such that the equation

$$\frac{(\ln x)^4}{x} = e^{-4}k$$

has **TWO DISTINCT** real roots.

30. [**5** points] Evaluate the following limit:

$$\lim_{n\to\infty} \sqrt[3]{n}\left(\sqrt[3]{n^2+2022n+1} - \sqrt[3]{n^2+1}\right)$$